Public-Private Mix of Health Expenditure: An OLG Political Economy Model and Quantitative Analysis*

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Abstract

This paper constructs a simple overlapping generations (OLG) model to examine decisions on public and private health spending under majority voting. In the model, agents with heterogeneous incomes decide how much to consume, save, and spend on health care, and vote for public health expenditure. Agents' survival probabilities are determined by a CES composite of public and private health expenditure. The existence and uniqueness of the voting equilibrium are established. A quantitative exercise is conducted for a sample of advanced democratic countries and shows that the predicted public-private mix of health expenditure matches the data fairly well.

JEL code: D7, H51, I1

Keywords: Public-private mix, Health expenditure, Majority voting, Overlapping generations model

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1 Introduction

Achieving a good health status in the overall population is one of the most important goals in every society. There are various factors that can improve health status and life expectancy. Among those most important factors are public and private expenditure on health care. The former includes expenditure on public health programs that provide public hospitals, immunization, disease control and diagnostic health screening, invest in new medical facilities, and promote healthy environment through, e.g., reducing air and water pollutions. The latter refers to private expenditure on healthy food, preventive medicines and vitamins, preventive and diagnostic health screenings, etc.

Although the total spending on public and private health care has been rising, and consequently life expectancy and health status have been improving in most countries, there are considerable variations in the mixture of public and private health spending over time and across countries. For instance, the share of public health in total health expenditure has increased by more than 10 percent since the 1970s in the U.S., Austria, Greece, and Japan, while it has decreased by more than 10 percent in Czech Republic, Norway and the UK. Across OECD countries, the share of public health ranges from less than 50 percent (e.g., the U.S.) to more than 80 percent (e.g., Denmark, Norway, Sweden, Japan, the UK, Czech Republic) in the 2000s.

Rapid population aging among OECD countries, partly due to the rise in life expectancy, has caused the finance of health care a major challenge. Many countries are undertaking or considering reforms to their health care systems, including privatising the public health care systems. Understanding the interaction between public and private health care and how the existing public-private mix of health expenditure is determined can help policy makers design or reform health care policies to achieve the best outcome.

This paper studies how the public-private mix of health expenditure is chosen by people in a democratic society collectively, when people can choose between public and private health spending in improving their life expectancies. We construct an overlapping generations model to explore how public and private health spending are determined by utility-maximizing agents with heterogenous incomes through majority voting, and how their decisions are shaped by the degree of substitutability between public and private health, income distribution and other economic factors, as well as preferences. Furthermore, the model is calibrated to conduct a quantitative exercise to investigate how well the model can explain the observed mixtures of health expenditure across a group of advanced democratic countries.

In the model, agents live for two periods: young adulthood and old adulthood. In young adulthood, agents receive exogenous heterogeneous incomes, and they decide how much to consume, to save for old adulthood, and to spend on health care, and they vote for the income tax to be used to finance public health expenditure. Agents' survival probabilities to old adulthood are endogenously determined by a CES composite of the public and private health expenditure. We establish the existence and uniqueness of the voting equilibrium as well as some qualitative properties of the equilibrium and derive the equations that implicitly determine the equilibrium majority choice of tax rate. Instructed by the equilibrium equations, we then calibrate the model and conduct a quantitative analysis.

The baseline values for parameters are calibrated to match moments of the Canadian data, including several important moments that capture the life expectancy, relative size and composition of health expenditure in Canada. The comparative static results suggest that the size of public health spending relative to national income and the share of public health in total health expenditure are quite sensitive to the degree of substitutability between private and public health and the parameter in the CES function that indicates the relative effectiveness of public and private health. We further infer these two parameters for each country using country-specific data, and construct the model predicted shares of public health in total health expenditure for each country in the sample. The results show that the predicted mixture of health expenditure matches the data quite well for the majority of countries, with an overall correlation of 0.41 between predicted shares of public health and corresponding data values. We then discuss several factors the model abstracts from that may have important implications for the public-private mix of health expenditure.

The contributions of this paper are two-fold. The first contribution is a theoretical one. Our paper is one of the few studies that aims to explain the mixture of public and private expenditure on health care. Our work is related to a theoretical literature on the coexistence of private and public provision of health care, e.g., see Epple and Romano (1996) and Gouveia (1997). Following the strand of literature on the socialization of commodities, they focus on the public provision of a private good –health care– through majority voting in a static micro-theoretic context. In their models, private and public health care are treated as perfect substitutes, and they directly enter the utility function as an ordinary consumption good. Our model also uses a voting mechanism to study the public and private mix of health expenditure.[1] We carry out the analysis in a dynamic macro-theoretic context that emphasizes the role of health care in improving life expectancy and allows for general substitutability between public and private health spending. We establish the existence and uniqueness of the voting equilibrium as well as some qualitative properties, in particular, a positive relationship between the majority choice of tax rate and the degree of income inequality.

Lahiri and Richardson (2008) develop a similar political economy model as ours in which individuals vote on the division of tax revenues between public health and a lump sum transfer payment. Their focus, however, is on how public and private health spending impact on wealth inequality in the long run. As the existence and uniqueness of a voting equilibrium cannot be analytically established, their analysis is primarily based on numerical simulations. Bethencourt and Galasso (2008) also present a political economy model that incorporates both public and private health spending, but they address the political complementarities between public health and social security. Another political economy model by Kifmann (2005) focuses on explaining the existence of public health insurance system in the absence of complete markets to insure against income risks.

[1] Our focus is on the public and private expenditure on health care, rather than the public and private provision of health care, although these two are closely linked.

This paper also relates to a large literature that incorporates health and endogenous mortality into a standard growth model to examine their implications for growth, poverty, inequality of income or wealth, and so on. Most of this literature considers one type of health expenditure, either private health in the forms of physical resources, time and human capital (Grossman, 1972; Blackburn and Cipriani, 2002; Leung, Zhang, and Zhang, 2004; Chakraborty and Das, 2005; and Castelló-Climent and Doménech, 2008) or public health (Chakraborty, 2004; and Ásia and Pueyo, 2006). A few studies consider the roles of both public and private health expenditure, such as Zhang, Zhang and Leung (2006), Tang and Zhang (2007), Bhattacharya and Qiao (2007), and Gupta and Vermeulen (2010). Again, the focus is on the developmental implications of health spending rather than the mixture of health spending. In particular, when public health is considered, the public policy involved (such as the tax rate) is exogenously given rather than being a collective choice. Besides, most of the studies with endogenous life-expectancy, such as Blackburn and Cipriani (2002), Chakraborty (2004), do not have disparities in health status across agents, while our model generates heterogeneous life-expectancy as private health spending varies with income, which is heterogenous.

The second contribution is a quantitative one. The aforementioned literature on health is largely purely theoretical. Our paper provides the first quantitative study on the public-private mix of health expenditure, and the quantitative results are reasonably good. A few country-specific empirical studies look at the substitutability between private and public health and our result is consistent with their findings. For example, our result implies that public and private health tend to be substitutable in the U.S. and complementary in the UK. Cutler and Gruber (1996a and 1996b) and Gruber and Simon (2008) study the effect of an increase in the coverage of Medicaid and Medicare in the U.S. on private health spending and find that public and private health are substitutes. Propper (2000) uses British Household Panel Survey between 1991 and 2000 and finds that private health appears to be complementary to public health. Finally, there is a relatively close match between the predicted shares of public health in total health spending and the corresponding data values for the group of 22 advanced democratic countries. These results suggest that our model provides a promising framework to study the determination of public and private health spending.

The quantitative results also provide useful information for empirical work on health. The model has identified several important factors for the size and composition of health spending, such as the substitutability between public and private health spending and their relative effectiveness in improving health outcome. These factors are largely ignored in the empirical literature on health, probably due to their unobservability in the data. Our quantitative exercise provides a way to infer the values for these important factors, which may be utilized in relevant empirical work.

Our study belongs to an emerging quantitative literature on health related issues, such as De Nardi, French, and Jones (2010), Jung and Tran (2010), and Hsu and Lee (2011). None of them, however, focuses on the public-private mix of health expenditure.

The rest of the paper proceeds as follows. Section 2 describes the model and establishes the analytical results. Section 3 presents the quantitative exercise, and Section 4 concludes.

2 The Model

In this section, we first describe the environment of the model economy, then solve the individuals' optimization problem, and finally characterise the majority voting equilibrium. All proofs are provided in the Appendix.

2.1 The Environment

We consider an overlapping generations economy with infinite number of periods. The economy is populated with a large number of agents who potentially live for two periods: young and old adulthood. In young adulthood, agents receive exogenous incomes and make decisions on their consumption and private health expenditure as well as saving for old age, and they vote for taxes that finance public health expenditure. Each young adult gives birth to one offspring, and thus the size of the young population is constant. In their old adulthood, agents simply consume what they have and exit the economy. All economic and political decisions are made in young adulthood.

Young adults in the same generation at time t are differentiated by their incomes, $y_{i,t}$, according to an exogenous probability distribution function $F_t(\cdot)$, where i refers to the ith young adult. The mean income at time t is thus given by $\bar{y}_t \equiv \int y dF_t(y)$.

Survival in young adulthood is certain, but survival in old adulthood endogenously depends upon an agent's health status in young adulthood. That is, we assume that young adult i's survival probability to old adulthood, $p_{i,t} \in (0,1)$, is a function of the health capital acquired in young adulthood, $X_{i,t}$:

$$p_{i,t} = p\left(X_{i,t}\right), \tag{1}$$

where $\partial p/\partial X_{i,t} > 0$, $\partial^2 p/\partial X_{i,t}^2 < 0$, $p(0) = 0$ and $\lim\limits_{X_{i,t}\to\infty} p_{i,t} = 1/\kappa < 1$. The health capital is defined as a CES composite of public and private health expenditure:

$$X_{i,t} = (\phi_H H_t^\rho + \phi_h h_{i,t}^\rho)^{1/\rho}, \tag{2}$$

where H_t denotes per capita public health expenditure in period t, $h_{i,t}$ is the private health expenditure of agent i in period t.[2] The parameters $\phi_H \in (0,1)$ and $\phi_h \equiv 1 - \phi_H$ indicate the effectiveness of public and private health expenditure in forming health capital, respectively, and $\rho \in [0,1]$ measures the elasticity of substitution between public and private health expenditure, which is a constant given by $\varepsilon \equiv 1/(1-\rho)$. The restriction that $\rho \in [0,1]$ is needed to establish the existence of a voting equilibrium in later sections. It implies that

[2] In their specification of health technology, Bhattacharya and Qiao (2007) and Gupta and Vermeulen (2010) consider a complementary relationship between private and public health expenditure in improving longevity.

public and private health spending are substitutable in contributing to an individual's health status, with the elasticity of substitution $\varepsilon \in [1, \infty)$. When $\rho = 0$ ($\varepsilon = 1$), the health capital takes the Cobb-Douglas form, and when $\rho = 1$ ($\varepsilon = \infty$), the health capital takes the linear form and public and private health spending are perfectly substitutable.

The health technology specified in (2) is a reduced-form representation of the functioning of public and private health care in a society. In practice, the structure of a health care system is shaped by various political, demographical, institutional, as well as economic factors. Differences in any of these factors can give rise to different structures and roles of public and private health care in the overall system. As a consequence, the interaction of public and private health care and how they combine to affect the health status of a society are complex issues. Endogenizing such interaction is beyond the scope of this paper. Instead, we use the reduced-form health technology in (2) to capture the contributions of private and public health spending in improving people's health status. This form allows for different effectiveness of public and private health spending as well as a general degree of substitutability between these two. When the parameters ϕ and ρ are calibrated for each country in the quantitative analysis, the different values reflect differences in the underlying factors that form each country's health care systems.

The assumption in (1) implies that an agent's mortality later in life is determined by her health status in young adulthood, which depends on her own choices of health expenditure. This assumption is extensively used in the literature that links health and longevity, such as Hall and Jones (2007) and Chakraborty and Das (2005).[3]

The lifetime utility of agent i at time t is defined over her consumption in young adulthood, $c_{i,t} \in R_+$, and consumption in old adulthood, $d_{i,t+1} \in R_+$:[4]

$$U_{it} = \ln(c_{i,t}) + \beta p_{i,t} \ln(d_{i,t+1}), \tag{3}$$

[3] An alternative assumption is to assume that an agent's mortality depends on her parents' decisions rather than her own, see Blackburn and Cipriani (2002) and Castelló-Climent and Doménech (2008).

[4] The utility from death is assumed to be zero. In our calibration, the values of $c_{i,t}$ and $d_{i,t+1}$are well above 1 such that the utilities from survival are positive.

where $\beta \in (0,1)$ is the subjective discount factor on expected utility from consumption in old adulthood. The budget constraints of agent i in young and old adulthood, respectively, are given by:

$$c_{i,t} + s_{i,t} + h_{i,t} = (1-\tau_t)y_{i,t}, \tag{4}$$

$$d_{i,t+1} = R_{t+1}s_{i,t}. \tag{5}$$

Agent i draws income $y_{i,t}$ from the exogenous distribution, pays income taxes at uniform rate τ_t, spends her disposable income on consumption in young adulthood, private savings, and private health spending. To deal with the mortality risk, we follow the strand of literature (e.g., Chakraborty, 2004) that assumes a perfectly competitive annuities market for private savings. That is the gross rate of return on private savings is given by $R_{t+1} \equiv (1+r_{t+1})/\bar{p}_t$, where $\bar{p}_t$ is the average survival probability, and $1+r_{t+1}$ is the exogenous gross interest rate. The average survival probability is endogenized in equilibrium (that is, when agents take $\bar{p}_t$ as given in their individual optimization, their decisions would lead to an average survival probability that is exactly equal to $\bar{p}_t$ in the equilibrium). In old adulthood, agent i simply consumes her private saving and any interest income earned.

Public health expenditure, H_t, is financed by income taxes collected from young adults in period t. Government budgets are balanced in every period:

$$H_t = \tau_t \bar{y}_t. \tag{6}$$

The tax rates prevailing in each period are endogenously determined by a majority voting mechanism. That is, in period t, each young adult votes on her preferred tax rate and the collective choice of the tax rate, τ_t, is determined by the majority rule. Note that in our setup, old adults do not have incentives to vote, as taxes are paid by young adults and health capital is formed during young adulthood as well.[5]

[5] We thank an anonymous referee for pointing this out.

2.2 Individual's Optimization for a Given Tax Rate

Consider the following parametric form for the survival probability $p_{i,t}$ that is strictly increasing and strictly concave in health capital:[6]

$$p_{i,t} = p\left(X_{i,t}\right) = \frac{X_{i,t}}{1+\kappa X_{i,t}},\ \kappa > 1, \tag{7}$$

and the CES form of health capital in (2) with $\rho \in [0,1]$. Agent i's survival probability depends positively on her choice of private health expenditure:

$$\frac{\partial p}{\partial h_{i,t}} = \frac{\phi_h X_{i,t}^{1-\rho} h_{i,t}^{\rho-1}}{\left(1+\kappa X_{i,t}\right)^2} > 0. \tag{8}$$

Agent i's utility maximization problem is to choose $c_{i,t}$, $d_{i,t+1}$, $s_{i,t}$ and $h_{i,t}$ to maximize (3) subject to (2), (4), (5), (6), and (7), taking as given R_{t+1}, τ_t, $y_{i,t}$ and $\bar{y}_t$.[7] The first-order conditions are given by:[8]

$$s_{i,t} : s_{i,t} = \frac{\beta p_{i,t}}{\left(1+\beta p_{i,t}\right)}\left((1-\tau_t)y_{i,t} - h_{i,t}\right) \tag{9}$$

$$h_{i,t} : -\frac{1}{(1-\tau_t)y_{i,t} - s_{i,t} - h_{i,t}} + \beta\frac{\partial p}{\partial h_{i,t}} \ln\left(R_{t+1}s_{i,t}\right) = 0. \tag{10}$$

Combining these two first-order conditions and Eq. (8) gives the equation that implicitly determines $h_{i,t}$ as a function of $y_{i,t}$, $\bar{y}_t$, R_{t+1}, τ_t, β, ρ, ϕ_H and κ:

$$\frac{\left(1+\beta p_{i,t}\right) h_{i,t}^{1-\rho}}{(1-\tau_t)y_{i,t} - h_{i,t}} = \frac{\beta\phi_h X_{i,t}^{1-\rho}}{\left(1+\kappa X_{i,t}\right)^2} \ln\left(R_{t+1}\frac{\beta p_{i,t}}{1+\beta p_{i,t}}\left((1-\tau_t)y_{i,t} - h_{i,t}\right)\right). \tag{11}$$

[6] A similar functional form for the probability of survival is assumed in Chakraborty (2004).

[7] By taking R_{t+1} as given, agents do not internalize the external effects of their decisions on the average survival probability and hence on the annuity return. Philipson and Becker (1998) argue that ignoring this externality will lead to excessive health expenditure. This issue may remain in our model, even though the average survival probability and hence the return on savings will be endogenized in equilibrium. There may exist other voting mechanisms that could mitigate this externality issue.

[8] The second-order condition for a local maximum also holds when $\partial s/\partial h_{i,t} < 0$. See the Appendix for more details.

For the ease of notation, we write the functions for optimal private health expenditure, health capital, private savings and survival probability as $h_{i,t} \equiv h(y_{i,t};\tau_t)$, $X_{i,t} \equiv X(y_{i,t};\tau_t)$, $s_{i,t} \equiv s(y_{i,t};\tau_t)$, and $p_{i,t} \equiv p(y_{i,t};\tau_t)$, respectively.

It is obvious from (9) that an increase in the probability of survival increases private savings, as young adults who expect to live longer are effectively more patient and more willing to save for old adulthood consumption. This result is consistent with empirical findings. Hurd, McFadden and Gan (1998) and Tsai, Chu and Chung (2000), for example, find evidence in household data that an increase in prospective longevity leads to higher saving rates.

Although there is no explicit solution for $h_{i,t}$ from (11), we are able to obtain some interesting analytical results. It is shown that the comparative static properties summarized in Proposition 1 hold under some reasonable assumptions (see the Appendix for details). These assumptions are (1) $\partial s/\partial h_{i,t} < 0$, (2) $\partial^2 p/\partial h_{i,t}\partial H_t < 0$, and (3) $\partial s/\partial \tau_t < 0$. Assumption (1) is sufficient to establish part (i) and (iii) of Proposition 1, and assumptions (2) and (3) are sufficient for establishing part (ii). For (i), note that an increase in private health spending has two effects on savings. A direct effect is to crowd out savings, and an indirect effect is to increase savings through a positive effect on the survival probability. So assumption (i) states that the direct negative effect dominates the indirect positive effect. Assumption (ii) states that the marginal benefit of private health expenditure on survival probability decreases with public health expenditure. And assumption (iii) states that the negative income effect of an increase in tax rate on savings dominates the positive indirect effect on savings through a higher survival probability resulting from a higher public health expenditure.[9]

Proposition 1 *Under reasonable assumptions, the following properties hold:*

(i) Private health expenditure $h_{i,t} \equiv h(y_{i,t};\tau_t)$ *increases with private income, that is* $\partial h/\partial y_{i,t} >$

[9] We have also numerically checked, using baseline parameter values as reported in Table 1, that these assumptions hold at all income levels for τ_t between 5 percent and 10 percent. This range of tax rate is chosen as the actual ratios of public health expenditure to national income, represented by τ_t in the model, are within this range for all countries in our data sample.

0.

(ii) Private health expenditure $h_{i,t} \equiv h(y_{i,t}; \tau_t)$ *decreases with income tax rate, that is* $\partial h / \partial \tau_t < 0$.

(iii) Individual's probability of survival $p_{i,t} \equiv p(y_{i,t}; \tau_t)$, *increases with private income, that is* $\partial p / \partial y_{i,t} > 0$.

Part (i) implies that private health is a normal good, as suggested by the empirical literature.[10] Private health spending is also affected by income taxes that finance public health spending. The negative relationship between private health and income tax rate in Part (ii) comes from two forces. First, a higher income tax rate reduces the disposable income. And second, a higher income tax rate leads to higher public health expenditure which crowds out private health expenditure, as private health and public health expenditure are substitutable with $\rho \in (0, 1)$. Part (iii) shows that given the tax rate, survival probabilities increase with private incomes because the probability of survival increases with private health spending and private health is a normal good. This result is consistent with the positive effect of income on life expectancy found in the empirical literature (e.g., see Preston, 1975; Pritchett and Summers, 1996; and Cutler et al, 2006, among others).

So far, we have characterised the individual's problem for a given tax rate. Next we characterise the preferred tax rate of each voter and the majority choice of tax rate under majority voting.

2.3 The Majority Choice of Tax Rate

Individuals take R_{t+1}, $y_{i,t}$ and parameters as given, and the preferred tax rate by agent i, denoted as $\tau_{i,t}$, maximizes her indirect utility $V_{i,t}$:

$$V_{i,t} = \ln \left(\frac{(1 - \tau_{i,t}) y_{i,t} - h(y_{i,t}; \tau_{i,t})}{1 + \beta p(y_{i,t}; \tau_{i,t})} \right) + \beta p(y_{i,t}; \tau_{i,t}) \ln \left(R_{t+1} s(y_{i,t}; \tau_{i,t}) \right), \tag{12}$$

[10] Many empirical studies find a positive correlation between income and private health expenditure (see e.g., Feinstein, 1993, for a review of the literature). Some other studies find that private health is a luxury good with income elasticity larger than one (see e.g., Scanlon, 1980; and Parker and Wong, 1997).

where $s_{i,t} \equiv s(y_{i,t};\tau_{i,t})$ is given in (9). Hence,

$$\frac{\partial V_{i,t}}{\partial \tau_{i,t}} = -\frac{(1+\beta p(y_{i,t};\tau_{i,t}))\left(y_{i,t}+\frac{\partial h(y_{i,t};\tau_{i,t})}{\partial \tau_{i,t}}\right)}{((1-\tau_{i,t})y_{i,t}-h(y_{i,t};\tau_{i,t}))} + \beta\frac{\partial p(y_{i,t};\tau_{i,t})}{\partial \tau_{i,t}}\ln(R_{t+1}s(y_{i,t};\tau_{i,t})).$$

The preferred tax rate of agent i is determined by $\partial V_{i,t}/\partial \tau_{i,t} = 0$, which simplifies to (see the proof of Proposition 2 in the Appendix):

$$H_{i,t} = \left(\frac{\phi_h}{\phi_H}\frac{y_{i,t}}{\bar{y}_t}\right)^{-\frac{1}{1-\rho}} h_{i,t}, \text{ where } h_{i,t} \equiv h(y_{i,t};\tau_{i,t}) \text{ and } H_{i,t} \equiv \tau_{i,t}\bar{y}_t. \tag{13}$$

Recall that $h_{i,t} \equiv h(y_{i,t};\tau_{i,t})$ is implicitly determined by (11) as a function of $y_{i,t}$ and $\tau_{i,t}$.

Given R_{t+1}, $y_{i,t}$ and parameters, an agent i's preferred tax rate, $\tau_{i,t}$, is unique because the left-hand side of (13) is strictly increasing in $\tau_{i,t}$ and the right-hand side is strictly decreasing in $\tau_{i,t}$ ($h_{i,t}$ is strictly decreasing in $\tau_{i,t}$ as stated in Part (ii) of Proposition 1). Proposition 2 shows that the preferred tax rates decrease with private incomes, i.e. $\partial\tau_{i,t}/\partial y_{i,t} < 0$, under the assumption $(\partial h/\partial y_{i,t})(y_{i,t}/h(y_{i,t};\tau_{i,t})) < 1/(1-\rho)$, which states that the income elasticities of private health spending is smaller than the elasticity of substitution between private and public health spending.[11] This negative relationship between agents' preferred tax rates and their incomes follows from the property that private health expenditure decreases with tax rates (Proposition 1 (ii)). It also comes from the redistributive role of public health across heterogenous agents within a generation. Because agents receive the same benefit from public health regardless of their income levels while the contributions are proportional to their private incomes, agents with lower incomes would prefer higher tax rates.

As the choice of voters is over a single dimension–the income tax rate, to show the existence of a majority choice of tax rate, we need to show that the indirect utilities, as defined in (12), are single-peaked in tax rates. Proposition 2 establishes the single-peakedness

[11] We have also numerically checked that this assumption holds at all income levels except very low income levels for tax rates ranging from 5 percent to 10 percent. Figure 1 shows that for the Canadian level of tax rate (6.83 percent), the majority of income elasticities of private health expenditure are less than $1/(1-\rho) = 4.845$.

of preferences in tax rates. Since the preferred tax rates are monotonically decreasing in voter's incomes, the majority choice of the tax rate is simply the preferred tax rate of the median voter (the young adult with median income), which is unique. Proposition 2 summarizes these results.

Proposition 2 *For a given return on savings R_{t+1}, there exists a unique majority choice of tax rate, $\tau_{m,t}$, which is the preferred tax rate of the young adult with median income.*

Therefore, for a given $\bar{p}_t$ and hence a given R_{t+1}, the unique majority choice of tax rate, $\tau_{m,t}$, satisfies $\partial V_{m,t}/\partial \tau_{m,t} = 0$, and hence it is implicitly determined by

$$\frac{h_{m,t}}{H_{m,t}} = \left(\frac{\phi_h}{\phi_H} \frac{y_{m,t}}{\bar{y}_t} \right)^{\frac{1}{1-\rho}}, \tag{14}$$

where $H_{m,t} = \tau_{m,t}\ \bar{y}_t$. Note that $h_{m,t}$ is implicitly determined by

$$\frac{(1+\beta p_{m,t})\, h_{m,t}^{1-\rho}}{(1-\tau_{m,t})y_{m,t} - h_{m,t}} = \frac{\beta \phi_h X_{m,t}^{1-\rho}}{(1+\kappa X_{m,t})^2} \ln \left(R_{t+1} \frac{\beta p_{m,t}}{1+\beta p_{m,t}} \left((1-\tau_{m,t})y_{m,t} - h_{m,t} \right) \right), \tag{15}$$

where the median survival probability, $p_{m,t}$, using the parametric form in (7), is given by

$$p_{m,t} = p\left(X_{m,t}\right) = \frac{1}{\kappa + \frac{1}{\left[\phi_H + \phi_h \left(\frac{h_{m,t}}{H_{m,t}}\right)^{\rho}\right]^{1/\rho} H_{m,t}}}. \tag{16}$$

So far, we have established that for a given average survival probability $\bar{p}_t$ and hence a given return on private savings $R_{t+1} \equiv (1+r_{t+1})/\bar{p}_t$, there exists a unique majority choice of tax rate, $\tau_{m,t}$ and hence a unique majority choice of public health expenditure given by $\tau_{m,t}\bar{y}_t$. In a voting equilibrium, $\bar{p}_t$ is endogenized. That is, when agents take $\bar{p}_t$ as given in their individual optimization and voting, the majority choice of public health expenditure and the optimal private health expenditures $h_{i,t}$'s (implicitly determined by (11) with τ_t given by the majority choice of tax rate $\tau_{m,t}$ corresponding to $\bar{p}_t$) would lead to an average survival probability that is exactly equal to $\bar{p}_t$. In the Appendix, we provide a discussion on the existence and uniqueness of such a $\bar{p}_t$. In the quantitative analysis, whenever we solve the model numerically we iterate on values of $\bar{p}_t$ until the equilibrium $\bar{p}_t$ is found (refer to the calibration procedure in Section 3.1). The numerical computation suggests that the equilibrium $\bar{p}_t$ exists and it is unique.

From now on, we assume that in our previous equations, $\bar{p}_t$ is the equilibrium average survival probability and hence R_{t+1} is the equilibrium return on private savings. Consequently, the majority choice of $\tau_{m,t}$ characterized by Eq. (14) to (16) is the unique equilibrium tax rate in the voting equilibrium.

2.4 Comparative Static Properties

Using Eq. (14) to (16), we can establish some comparative statics results concerning the size of public health relative to national income, indicated by $\tau_{m,t}$, and the public-private mixture in health expenditure, indicated by the ratio of median private health expenditure to per capita public health expenditure, $h_{m,t}/H_{m,t}$. With the equilibrium characterized by Eq. (14) to (16), the following properties hold: (i) the equilibrium tax rate $\tau_{m,t}$ decreases and $h_{m,t}/H_{m,t}$ increases with $y_{m,t}/\bar{y}_t$, given $\bar{y}_t$; (ii) $h_{m,t}/H_{m,t}$ decreases with ϕ_H; (iii) $h_{m,t}/H_{m,t}$ decreases (increases, does not change) with ρ when $(\phi_h/\phi_H)(y_{m,t}/\bar{y}_t)$ is less than one (greater than, equal to) one.

Property (i) implies that when income inequality is higher (lower $y_{m,t}/\bar{y}_t$), a majority of voters tend to favor a higher income tax rate or a higher level of public health as well as a lower ratio of private health to public health expenditure. This result is consistent with a large literature that models majority voting over the tax rate; see Meltzer and Richard (1981) and Krusell and Rios-Rull (1999) for examples. In those models, tax revenues are used to finance some type(s) of redistributive expenditure such as transfer payments and pensions. In our model, public health, financed by tax revenues, also plays a redistributive role within a generation. Property (ii) is self-intuitive, as a higher value of ϕ_H implies a more effective role of public health expenditure. Property (iii) implies that when the relative effectiveness of private health to public health (ϕ_h/ϕ_H) is not too big ($y_{m,t}/\bar{y}_t$ is typically less than 1 in the data), a majority of voters would be more likely to substitute public health for private health expenditure with a higher elasticity of substitution (a higher value of ρ).

It is difficult to derive further analytical results. Instead, we calibrate the parameters and conduct a quantitative analysis for a sample of advanced democratic countries.

3 Quantitative Exercise

3.1 Baseline Calibration

Parameters of the model include: the discount factor (β), the interest rate (r), the parameter in the parametric form of survival probability (κ), the parameter measuring the relative effectiveness of public and private health in the CES function of health capital (ϕ_H, $\phi_h = 1-\phi_H$), and the parameter measuring the degree of substitution between public and private health (ρ). We calibrate these parameters to match certain characteristics of the Canadian data (2000-2009). Canada is chosen for its well-established universal public health care system as well as data availability. To simplify notation, we drop subscript t.

First, we set a period in the model to be 30 years and young adulthood starts at age 31, that is, young adulthood is from 31 to 60 years old, and old adulthood is from 61 to 90 years old. Then β and r can be set to match the average annual real interest rate in Canada, which is around 2.51% according to *World Development Indicator-2011*. That is, $r = (1 + 0.0251)^{30} - 1$, and $\beta = 1/(1 + r)$.

The parameters κ, ρ and ϕ_H are not directly deducible from the data. We calibrate them jointly using the three equations that characterize the solution of the model, namely, Eq. (14)-(16). Note that the value of public health expenditure H_m also needs to be calibrated, as it is not unit-free and cannot be determined from the data.[12] We calibrate κ, ρ, ϕ_H, and

[12] The calibrated value of H is later used to determine a scale factor to scale down average incomes across countries.

H to match four moments of the Canadian data.

The first moment is the average median survival probability, p_m. In the model $p_{m,t}$ and $h_{m,t}$ refer to the survival probability and the private health expenditure of the median voter (the individual who has the median income) in period t. Recall that private health is a normal good and the survival probability is strictly increasing in private health expenditure, so $h_{m,t}$ and $p_{m,t}$ are the median private health expenditure and median survival probability in period t as well. Using the data from *World Health Statistics - 2011*, we find that the median age at which people would die, conditional on that they have survived over 60 years but under 90 years, is 83.526. So p_m is set to $(83.526 - 60)/(90 - 60) = 0.784$.[13]

The second moment is the ratio of average annual public health to national income, represented by $\tau_m \equiv H_m/\bar{y}$ in the model, which is 6.83% according to *OECD Health Dataset - 2011*. The third moment is the average annual share of public health expenditure in total health expenditure, $H_m/(H_m + \bar{h})$, where $\bar{h}$ denotes the average private health expenditure.[14] This moment is equal to 70.14 percent for Canada according to *OECD Health Dataset - 2011*. The fourth moment is the ratio of median to mean private health expenditure, $h_m/\bar{h}$, which is 0.6962 according to *National Household Survey* from *Statistics Canada*.[15] Besides, the income inequality measure that is taken as exogenous, $y_m/\bar{y}$, is equal to 0.8659 according to *OECD.Stat Extracts - 2011*. Given the moments above, the ratio of median private health expenditure to per capita public health, h_m/H_m, appearing in Eq. (14) to (16), is obtained as $h_m/H_m = \left(h_m/\bar{h}\right)\left(\bar{h}/H_m\right)$.

[13]The calculations are done using countries' life-tables provided in the data. A life-table considers a sample of $100,000$ people and reports death rates for each age category (less than 1, 1-4 years, 5-9, ..., 95-100, 100+). In calculating p_m, we also considered a maximum age of 95 and 100 rather than 90 years, and found that p_m does not change significantly.

[14]Total expenditure on health is defined by OECD Health Data 2011 as the sum of public and private health expenditure. Public expenditure on health is health expenditure incurred by public funds. This includes promoting health and preventing disease, curing illness, caring for persons require nursing care, publicly-financed gross capital formation in health facilities plus capital transfers to the private sector for hospital construction and equipment, expenditure on medical goods. Private expenditure on health care is privately funded part of total health expenditure. Private sources of funds include out-of-pocket payments (both over-the-counter and cost-sharing), private insurance programmes, charities and occupational health care.

[15]National Household Survey is not publicly available and is purchased from Statistics Canada. Available at: http://cansim2.statcan.gc.ca/

The calibration procedure is as follows. For a given H_m and $\bar{p}$ (such that $R = (1+r)/\bar{p}$), κ, ρ and ϕ_H are solved jointly from (14)-(16). We assume that the distribution of income is a log-normal distribution with parameters μ and σ, and calibrate μ and σ to match the mean income and income inequality in the Canadian data, that is, $\mu = \ln(\bar{y} \cdot (y_m/\bar{y}))$, where $\bar{y}$ is the scaled mean income given by H_m/τ_m, and $\sigma = \sqrt{2\ln(\bar{y}/y_m)}$. Then we draw $20,000$ income realizations from this distribution, and for each income draw, $y_{i,t}$, we solve the corresponding private health expenditure, $h_{i,t} = h(y_{i,t}; \tau_{i,t})$, from (11) to get $p_{i,t}$. A new value of $\bar{p}$ is calculated as the average of $p_{i,t}$'s. If it is different from the $\bar{p}$ given, we use this new value and repeat the process described above until $\bar{p}$ converges. Once $\bar{p}$ converges, we check whether the computed value of $H_m/(H_m + \bar{h})$, where $\bar{h}$ is the average of $h_{i,t}$'s solved corresponding to each income draw, matches the share of public health in total health expenditure in the data. If they are different, another value of H_m is chosen and the whole process is repeated, until the computed $H_m/(H_m + \bar{h})$ matches its data counterpart.[16]

Table 1: Baseline Cabibration

Parameters	Description	Calibrated Value
β	Discount factor	$1/(1+2.51\%)^{30}$
κ	Parameter in the survival probability	1.1784
ρ	Degree of substitution between public and private health	0.7936
ϕ_H	Effectiveness of public health in health production	0.5267
H	Public health expenditure	16
Moments	Description	Data Value
r	Interest rate	2.51%
$p_{m,t}$	Median survival probability	0.784
$\tau_m \equiv H_m/\bar{y}$	Ratio of public health to national income	6.83%
$H_m/(H_m+\bar{h})$	Share of public health in total health expenditure	70.14%
$h_m/\bar{h}$	Ratio of median to mean private health expenditure	0.6962
$y_m/\bar{y}$	Ratio of median to mean income	0.8659

Table 1 summarizes the calibrated parameter values and the data moments used for calibration. The calibration implies that the maximum survival probability to age 90, which would be achieved when the health capital approaches infinity, is given by $1/\kappa = 0.85$. The

[16]The computed $H_m/(H_m + \bar{h})$ is 72.23%, close to its data value 70.14%. All other moments are exactly matched.

calibrated value of ρ implies an elasticity of substitution between public and private health of 4.85, suggesting that public and private health spending are substitutable to a relatively high degree. The value of ϕ_H is a bit higher than 0.5, suggesting that public health spending is slightly more effective than private health spending in contributing to the society's health capital. Due to the lack of relevant empirical or quantitative studies, we cannot compare the calibrated values for ρ and ϕ_H with other studies. Nevertheless, these values seem realistic for the Canadian economy. Finally, the value of H, which works like a scale factor, is calibrated to be 16.

Under the baseline calibration, the income elasticities of public health and total health expenditure with respect to national income are computed as 0.69 and 0.74, respectively, suggesting that health care is not a luxury good at aggregate level. At the individual level, the income elasticities of private health expenditures (h_i) with respect to individual incomes (y_i) are 2.09 on average (see Figure 1 for the distribution of individual income elasticities), suggesting that private health care is a luxury good for individuals.[17] The income elasticity of health expenditure has been an important subject addressed in the empirical literature. Our quantitative results are in line with empirical findings. For instance, the income elasticity of public health expenditure for Canada is estimated to be 0.77 by Di Matteo and Di Matteo (1998), and studies on private medical care like eyeglasses, plastic surgery, and nursing home care find income elasticities that are substantially greater than one (see e.g., Scanlon, 1980; and Parker and Wong, 1997).

In our model, private health expenditure and hence the probability of surviving to old adulthood are heterogeneous across individuals. Most existing studies that model endogenous life-expectancy, such as Blackburn and Cipriani (2002), Chakraborty (2004) and Hall and Jones (2007), do not have disparities in health status across agents. Based on $20,000$ income draws, Figure 1 plots the kernel density and cumulative distribution functions for

[17] To compute the income elasticities, we numerically calculate the derivatives of public health expenditure and total health expenditure with respect to average income. To compute the individual income elasticities, we numerically calculate the derivatives of individual private health expenditures with respect to individual incomes for $20,000$ random income draws.

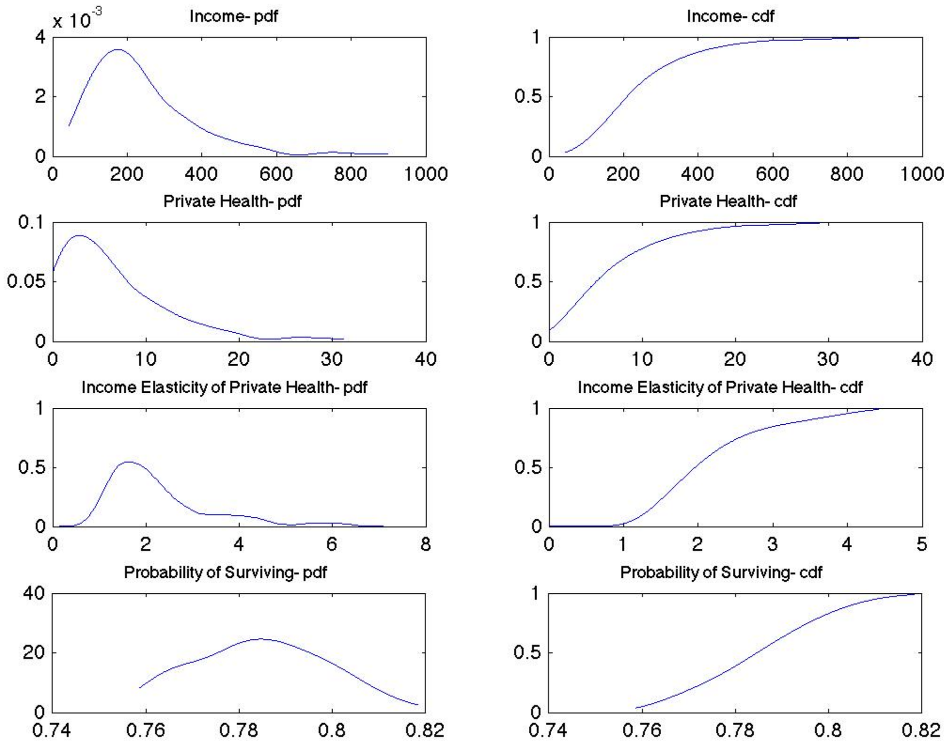

Figure 1: Kernel Density and Cumulative Distribution Functions for Income, Private Health Expenditure, Income Elasticity of Private Health and Survival Probability

income, private health expenditure, income elasticities of private health expenditure and survival probabilities in equilibrium. A notable feature from the figure is that the distribution of private health expenditure is much more skewed than the distribution of income, with about a quarter of the population having zero or close to zero private health expenditure. The distribution of income elasticities of private health expenditure is also much more skewed than the distribution of income and for most individuals private health is a luxury good with income elasticity above one. However, the distribution of survival probabilities appears quite symmetric, due to the contribution of public health. These qualitative features are broadly consistent with the data, though we do not have individual level data to conduct a quantitative comparison.

3.2 Sensitivity Analysis

Next, we conduct a numerical exercise to investigate the sensitivity of endogenous variables to variations in the primitives of the model. The aim is to see how the majority choice of tax rate, which in the model measures the size of public health relative to national income, and the public-private mix of health expenditure respond to variations in the primitives of the model, in particular, how sensitive they are to each variation.

Specifically, we examine how the majority choice of tax rate, τ_m, the average survival probability, $\bar{p}$, the ratio of median private health to public health expenditure, h_m/H_m, and the share of public health in total health expenditure, $H_m/(H_m + \bar{h})$, respond to changes in parameters β, κ, ϕ_H , ρ, and the statistics that characterise the distribution of income, $y_m/\bar{y}$ and $\bar{y}$. Baseline values for these parameters are the ones described in the calibration above. We consider 5, 10 and 15 percent variations of each parameter around its baseline value, with all other parameters kept at their baseline values. For each variation, h_m/H_m is determined by (14), and τ_m is solved from (14) to (16). To get the average survival probability $\bar{p}$ and the average private health expenditure $\bar{h}$, we follow the same procedure as in the baseline calibration, then the share of public health in total health spending is calculated as $H_m/(H_m + \bar{h})$, where H_m equals the product of computed τ_m and $\bar{y}$.

Table 2 summarizes the results from the numerical exercise. It is found that the ratio of per capita public health to average income, τ_m, decreases with κ, $y_m/\bar{y}$, and $\bar{y}$, and increases with β, ρ and ϕ_H. In terms of the magnitude of change, τ_m is most sensitive to variations in ϕ_H, also sensitive to variations in ρ and $y_m/\bar{y}$, and less sensitive to β, κ, and $\bar{y}$. The average survival probability $\bar{p}$ increases with β, ρ, ϕ_H, and $\bar{y}$, decreases with κ, and has no clear relationship with $y_m/\bar{y}$. In terms of the magnitude, $\bar{p}$ is most sensitive to κ and $\bar{y}$. The ratio of median private health to per capital public health spending, h_m/H_m, does not vary with β, κ, and $\bar{y}$, while decreases with ρ and ϕ_H and increases with $y_m/\bar{y}$. It is most sensitive to ϕ_H, and also quite sensitive to variations in ρ and $y_m/\bar{y}$. The share of public health in total health expenditure, $H_m/(H_m + \bar{h})$, increases with β, ρ, and ϕ_H, and decreases with $y_m/\bar{y}$. It is quite sensitive to ϕ_H and ρ followed by $y_m/\bar{y}$, while not sensitive to β, κ, and $\bar{y}$. Note that the analytical results for comparative statics stated in Section 2.5 are all confirmed by the numerical results.

Brief intuitions are as follows. A higher discount factor, β, implies that individuals care more about their old-age utility, so they prefer higher public health expenditure and hence a higher tax rate in order to have a higher average survival probability. An increase in κ implies a lower average survival probability at any given level of health capital, so individuals tend to vote for a lower level of public health. A higher ρ implies a greater substitutability between public and private health. With a ϕ_H greater than 0.5, individuals tend to substitute public health for private health, and so vote for a higher tax rate, or in other words, public health crowds out private health when ρ increases. Consequently, the ratio of median private health to public health expenditure falls and the share of public health in total health expenditure and the average survival probability rise. A higher ϕ_H indicates that the effectiveness of public health rises relative to private health in the formation of health capital and thus, individuals vote for a higher tax rate, which leads to a lower ratio of median private to public health and a higher share of public health as well as a higher average survival probability.

Table 2: Sensitivity of Variables to Parameters

	β	κ	ρ	ϕ_H	$\frac{y_m}{\bar{y}}$	$\bar{y}$
Majority Choice of Tax Rate (τ_m)						
-15%	0.0634	0.0773	0.0601	0.0359	0.0773	0.0721
-10%	0.0652	0.0741	0.0622	0.0477	0.0745	0.0707
-5%	0.0668	0.0710	0.0649	0.0590	0.0716	0.0695
Baseline	0.0683	0.0683	0.0683	0.0683	0.0683	0.0683
$+5\%$	0.0698	0.0657	0.0727	0.0750	0.0648	0.0672
$+10\%$	0.0712	0.0634	0.0784	0.0791	0.0612	0.0661
$+15\%$	0.0725	0.0612	0.0847	0.0812	0.0575	0.0651
Average Survival Probability ($\bar{p}$)						
-15%	0.7799	0.9199	0.7831	0.7804	0.7847	0.7777
-10%	0.7814	0.8699	0.7833	0.7809	0.7845	0.7802
-5%	0.7829	0.8250	0.7837	0.7823	0.7844	0.7823
Baseline	0.7843	0.7843	0.7843	0.7843	0.7843	0.7843
$+5\%$	0.7855	0.7477	0.7852	0.7864	0.7846	0.7862
$+10\%$	0.7867	0.7142	0.7867	0.7884	0.7854	0.7879
$+15\%$	0.7877	0.6836	0.7888	0.7902	0.7863	0.7896
Ratio of Median Private Health to Per Capita Public Health (h_m/H_m)						
-15%	0.2964	0.2964	0.4625	1.3768	0.1349	0.2964
-10%	0.2964	0.2964	0.4155	0.8236	0.1779	0.2964
-5%	0.2964	0.2964	0.3607	0.4941	0.2312	0.2964
Baseline	0.2964	0.2964	0.2964	0.2964	0.2964	0.2964
$+5\%$	0.2964	0.2964	0.2219	0.1773	0.3755	0.2964
$+10\%$	0.2964	0.2964	0.1386	0.1054	0.4705	0.2964
$+15\%$	0.2964	0.2964	0.0565	0.0621	0.5835	0.2964
Ratio of Public to Total Health Spending ($H_m/(H_m+\bar{h})$)						
-15%	0.7215	0.7222	0.6442	0.3979	0.7787	0.7208
-10%	0.7220	0.7225	0.6646	0.5179	0.7610	0.7213
-5%	0.7221	0.7223	0.6903	0.6298	0.7449	0.7229
Baseline	0.7223	0.7223	0.7223	0.7223	0.7223	0.7223
$+5\%$	0.7226	0.7224	0.7627	0.7914	0.6960	0.7228
$+10\%$	0.7228	0.7225	0.8110	0.8405	0.6646	0.7224
$+15\%$	0.7229	0.7224	0.8613	0.8753	0.6300	0.7228

The relationship between τ_m and $y_m/\bar{y}$ is standard: lower income inequality leads to a lower preferred tax rate. Hence, following an increase in $y_m/\bar{y}$, the ratio of median private health to public health expenditure increases and the share of public health decreases. The negative relationship between $\bar{y}$ and τ_m shows that the majority choice of tax rate decreases with a society's average income, suggesting that the society can meet its finance needs with a lower tax rate as it becomes richer, given that income inequality is unchanged. The positive effect of $\bar{y}$ on the average survival probability is in line with the observation that wealthier countries have higher life expectancy than poorer countries, as well as with the empirical literature. For instance, Preston (1975) finds that average income contributes positively to life expectancy, and Kennelly et al. (2003) provide evidence that per capita income and public health expenditure are both positively associated with improved health outcomes. There are some other studies that find a strong positive effect of wealth on life expectancy (see, e.g., Deaton and Paxson, 2001 and Attanasio and Emmerson, 2003).

3.3 Cross-Country Analysis

As described in the Introduction, there are considerable differences in the public-private mix of health expenditure across OECD countries. Instructed by the model, we conduct some cross-country quantitative exercises to explore what might account for the differences in the mixture of health expenditure for a sample of 22 OECD countries with the highest index of democracy.[18] These countries have relatively similar economic and political backgrounds.

First, we want to examine the role of income distribution, including average income as well as income inequality, in accounting for the observed differences in the mixture of health expenditure.[19] We assume that countries only differ in their average incomes (scaled $\bar{y}$ using

[18] *Polity IV* dataset provides an index of democracy for all countries. This index is between 0 and 10. Our sample includes OECD countries with the highest index of democracy (9 and 10). However, not all countries with the index of 9 and 10 are included in our sample; because of data limitations. Figure 2 shows all countries in our sample.

[19] Income has traditionally been viewed, also confirmed by many empirical studies (e.g., Newhouse, 1977 and Hitiris and Posnett, 1992), as one of the most important determinants of total health expenditure, but there are very few empirical studies that investigate how income affects the mixture of health expenditure.

the same scaling factor implied for Canada) and ratios of median to mean income ($y_m/\bar{y}$), with all other parameters taking the same baseline values as calibrated for Canada. We then calculate the predicted shares of public health in total health expenditure for each country in the same way as described in the calibration and compare them with the corresponding data values. The data for $y_m/\bar{y}$ and $\bar{y}$ (PPP-based per capita GDP Constant 2000) for each country are obtained from *OECD.Stat Extracts - 2011* and *World Development Indicator-2011*, respectively.

This quantitative exercise shows that the predicted shares of public health in total health do not exhibit much variation across countries, and nor are they close to their data counterparts. Hence, income distribution does not seem to play an important role in accounting for the observed differences in the public-private mix of health expenditure.[20]. This finding should be interpreted with caution, since we only consider differences in the mean and variance of income distributions and ignore variations in all other factors across countries. A possible reason for this result is that the parameter β, κ, ϕ_H, ρ take the same values for all countries as Canada. In particular, the comparative static results highlight that the mixture of health expenditure is very sensitive to the degree of substitutability between public and private health spending, measured by ρ, and the relative effectiveness of public health, measured by ϕ_H. Another possible explanation is that in the model income is exogenously given such that there is no feedback effects between income and health expenditure: an individual with higher income spends more on health care and higher health spending leads to higher productivity and income.

In next quantitative exercise we calibrate ρ and ϕ_H for each country in the sample and

One exception is Di Matteo (2000), which studies the public-private mix of health expenditure in Canada and finds that an important determinant of the split is the share of individual income held by the top quantile of the income distribution–a measure of income inequality.

[20]We also conduct another two exercises. In one, we keep $y_m/\bar{y}$ fixed at its baseline value, while use country-specific $\bar{y}$ to find out the implied shares of public health, and in the other, we keep $\bar{y}$ fixed at its baseline value while varying $y_m/\bar{y}$. The results show that variations in $H_m/(H_m + \bar{h})$ have a weak positive relationship with $\bar{y}$ and a negative relationship with $y_m/\bar{y}$. This is consistent with the comparative static properties discussed earlier, however, the variations in $H_m/(H_m + \bar{h})$ induced by variations in income distributions are not big enough to account for the observed variations in the data.

re-compute the public-private mix of health expenditure predicted by the model. The comparative static results suggest that the share of public health in total health expenditure is not sensitive at all to β and κ. Thus, in this exercise we still keep β and κ at their baseline values for all countries and use country-specific $\bar{y}$ and $y_m/\bar{y}$ as in the previous exercise, while calibrating ρ and ϕ_H for each country to match two country-specific moments: the ratio of public health to national income ($H_m/\bar{y} \equiv \tau_m$), and the median survival probability (p_m). Given β and κ values, ρ and ϕ_H values for a particular country are solved jointly from Eq. (15) and (16), with h_m/H_m given by (14) and that country's $y_m/\bar{y}$ and scaled $\bar{y}$. Then the shares of public health in total health spending for each country are computed similarly as before. The implied income elasticities of total health expenditure are also computed for each country.

Table 3 reports for each country the calibrated values of ϕ_H and ρ, the ratios of predicted shares of public health in total health expenditure ($H_m/(H_m + \bar{h})$) to the corresponding data values, and the ratios of matched tax rates (τ_m) and median survival probabilities (p_m) to their data counterparts. Note that for all countries, except Czech Republic, Hungary and France, both ratios of matched tax rates and median survival probabilities to their data counterparts deviate from 1 by less than 1 percent, implying that these two moments are exactly matched in the calibration of ϕ_H and ρ. While for Czech Republic, Hungary and France, at least one ratio deviates from 1 by more than 9 percent. So we will exclude these three countries in our discussion of the results.

According to Table 3, the calibrated values of ϕ_H are higher than 0.6 for most countries in the sample. Finland, Italy, Japan, New Zealand, Portugal, Sweden, Switzerland and the UK are the nine countries having high ϕ_H (higher than 0.6), suggesting that in these countries public health plays a more effective role in forming the health capital of the society to promote life expectancy, while the U.S. has the lowest value of ϕ_H (around 0.45).

The calibrated values of ρ vary substantially across countries, ranging from 0.001 to 0.999, implying that the elasticities of substitution between private and public health expenditure,

ε, vary between 1 and 1,000 in the sample. However, except Sweden, the values of ρ for all other countries are either close to zero or close to 1. When ρ is close to zero or equivalently ε is close to one, public and private health spending are complementary in contributing to a society's health capital. Such countries include: Australia, Finland, Greece, Italy, Japan, New Zealand, Portugal, Spain, Switzerland and the UK. This result suggests that for these countries a Cobb-Douglas form can be a good approximation for the production function of health capital. On the contrary, when ρ is close to one, public and private health spending are highly substitutable. These countries include Austria, Denmark, Germany, Ireland, Netherlands, Norway and the U.S., suggesting that for these countries a linear form may be a good approximation for the health technology.

Table 3: Calibrated Values of Parameters and Computed Ratios of Moments to their Data Counterparts

Country	ϕ_H	ρ	$\left.\frac{\text{Model}}{\text{Data}}\right\vert_{H_m/H_m+\bar{h}}$	$\left.\frac{\text{Model}}{\text{Data}}\right\vert_{\tau}$	$\left.\frac{\text{Model}}{\text{Data}}\right\vert_{p_m}$
Australia	0.5940	0.0010	0.8993	0.9999	0.9922
Austria	0.4713	0.9990	1.0428	1.0079	1.0054
Canada	0.5264	0.7950	1.0285	0.9998	1.0000
Czech Republic	0.4646	0.9990	0.6343	1.0046	1.1443
Denmark	0.4858	0.9990	0.9533	0.9980	1.0067
Finland	0.6167	0.0010	0.8521	1.0002	0.9943
France	0.7907	0.3650	1.1184	0.9044	1.0000
Germany	0.5558	0.8450	1.0805	1.0002	1.0000
Greece	0.5176	0.0010	0.8786	1.0001	0.9716
Hungary	0.4631	0.2700	0.6616	0.9998	1.1505
Ireland	0.4645	0.9990	0.8095	1.0018	1.0040
Italy	0.6836	0.0010	0.9129	1.0000	0.9846
Japan	0.7057	0.0010	0.8691	0.9998	0.9909
Netherlands	0.4750	0.9990	0.9040	0.9926	1.0014
New Zealand	0.6677	0.0010	0.8623	1.0003	0.9827
Norway	0.4709	0.9990	0.9849	0.9715	1.0102
Portugal	0.6465	0.0010	0.9092	1.0001	0.9682
Spain	0.5732	0.0010	0.8148	1.0001	0.9809
Sweden	0.6761	0.5400	1.0075	0.9997	1.0000
Switzerland	0.7368	0.0010	1.2685	1.0004	0.9991
UK	0.6985	0.0050	0.8798	1.0001	1.0000
U.S.	0.4502	0.9990	1.6403	1.0085	1.0122

The results above also show that there is a high negative correlation, -0.7, between the calibrated values of ϕ_H and ρ. This suggests that for countries that have higher ϕ_H (i.e., public health is relatively more effective than private health), public and private health are more complementary, while for countries that have lower ϕ_H, public and private health are more substitutable. This result seems reasonable. When public health is more (less) effective, the majority of people in a society tend to use private health as a complement (a substitute) for public health in promoting their health status.

Recall that ρ and ϕ_H indicate the degree of substitutability between public and private health and the relative effectiveness of public vs. private health in improving health outcome. These two parameters capture the interaction of public and private health care in a society, which is a very complex issue in real life. Knowing about them can help policy makers design and evaluate health care policies. For instance, if the elasticity of substitution between public and private health is high, i.e., the two types of health expenditure are more substitutable, an increase in one type of health expenditure is more likely to crowd out the other type of health expenditure. So any proposed policy change or reform to the financing of health care should take this crowding out effect into consideration.

Our calibration exercise provides a way to infer the values of ρ and ϕ_H for each country. In particular, our results suggest that either a Cobb-Douglas form or a linear form is reasonable for the health technology in the majority of the countries. This finding could be utilised in future empirical or quantitative work. As there is little empirical work that looks at the interaction between public and private health, we are not able to assess whether our estimates are in line with the existing institutional arrangements of the health care systems in each country. However, there are some country-specific empirical studies which provide some empirical support for our estimates. For instance, our result implies that public and private health tend to be complementary in the UK and substitutable in the U.S. This is consistent with some empirical findings. Propper (2000) examines the choice between public and private health care using the British Household Panel Survey between 1991 and 2000, and finds that private health appears to be complementary to public health. Cutler and Gruber (1996a and 1996b) and Gruber and Simon (2008) study the impact on private health insurance of an increase in the coverage of Medicaid and Medicare in the U.S., and find that the increase in public health coverage has crowded out private health insurance substantially, suggesting that public and private health are substitutes.

The fourth column of Table 3 reports the ratio of predicted shares of public health in total health expenditure to their data counterparts for each country. Further, Figure 2 plots the predicted versus actual shares of public health for each country, where the solid line in the figure corresponds to the 45-degree line. As shown in Table 3 and Figure 2, our model predicts the best for 8 out of 21 countries, including Austria, Denmark, Germany, Italy, Netherlands, Norway, Portugal and Sweden (Canada is excluded as it is the benchmark country with a perfect match). For these countries the ratios of predicted shares to actual values are between 0.9 and 1.1. The model also predicts reasonably well for countries such as Australia, Finland, Greece, Ireland, Japan, New Zealand, Spain and the UK, with the ratios higher than 0.8 or lower than 1.2, and the prediction is not bad for Switzerland as well. The worst prediction is for the U.S. as the model over-predicted the share of public in total health spending. The correlation between the predicted and actual shares of public health is 0.41 if we exclude France, Hungary and Czech Republic from the sample given that the moments are not closely matched for these countries in the calibration. If we further exclude the U.S., the correlation becomes 0.63.

We consider the U.S. to be an outlier in the quantitative analysis. Among the countries in the sample, the U.S. has the highest total health expenditure relative to GDP (15 percent of GDP, on average, in 2000s) and the lowest share of public health in total health expenditure (45 percent). Public health is treated as a publicly financed universal health care system in the model, and the U.S. is the only country in the sample that does not have a universal public health care system. The two main public health programs in the U.S., Medicare and Medicaid, only provide coverage to particular groups of people: Medicare is for people who

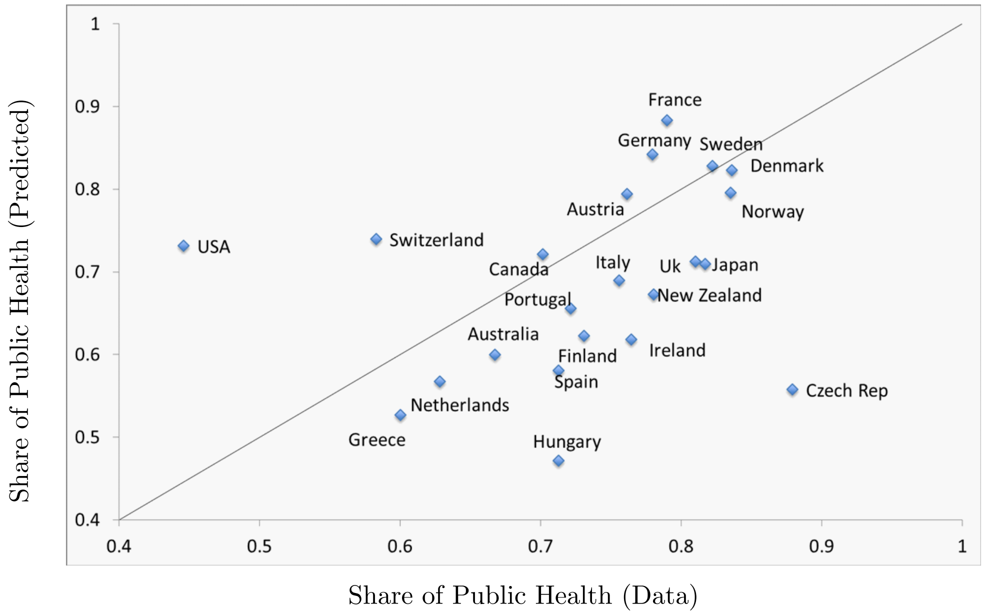

Figure 2: Shares of Public Health in a Sample of Democratic Countries (Predicted versus Data)

are above 65 years of age or permanently disabled, and Medicaid is for low-income families.

3.4 Discussion

The gap between the predicted public-private mix of health expenditure and the data may be due to many other factors that the model abstracts from. Countries differ in many factors that may potentially shape the structure of their health care systems, including institutional and demographical differences, as well as differences in people's preferences for various types of health services and in the pricing of health care services. For example, different countries utilize different policies, regulations, and market mechanisms (such as outsourcing of public health care and public-private partnerships) for health care, and hence public and private sectors play diverse roles in funding and providing health care across countries. As a consequence, the interaction of public and private health in the overall health care system is a complex issue. Indeed, a taxonomy of health care systems across countries is a difficult task, as discussed in Joumard, André and Nicq (2010).

Due to the complexity of the issue, an extensive discussion is beyond the scope of this paper. Nevertheless, next we discuss several factors that may be important for the public-private mix of health expenditure but are not considered in our model, including the demographic structure of the population, the structure of public financing and private financing of health care, as well as the pricing of health care services.

The demographic structure of the population may have important implications for the composition of health care spending. Health expenditure is typically highly concentrated, in particular, old people account for a much larger fraction of total health spending relative to their share in the population. So an aging population implies a higher demand for health care services, especially for public long-term health care. In fact, for countries in our sample, most of which have an aging population, there is a positive correlation (0.22) between the shares of the population above 65 years of age and the shares of public health in total health expenditure.

The focus in this study is on the composition of health expenditure: public versus private. We abstract from the different types of public and private financing of health care, as observed in the data. According to *OECD Health Dataset - 2011*, government revenues and social insurance are the two main sources of finance for public health care and out-of-pocket and private health insurance are the two main sources of private funding of health care. Figure 3 compares the compositions of public and private financing of health care across countries in the sample. It is clear that the composition of public financing differs substantially across countries. For some countries (e.g., Denmark, Australia, and Ireland) government revenue accounts for more than 95 percent of public financing, while for some other countries (e.g., Netherlands, France and Germany) public health care relies mostly on social insurance based funding. For the financing of private health care, the out-of-pocket contributions range from 30 percent to almost 100 percent. Understanding these differences within each type of financing of health care is important for us to understand the observed differences in the public-private mix of the overall expenditure on health care across countries.

Last but not least, the pricing of health care services also has important implications for

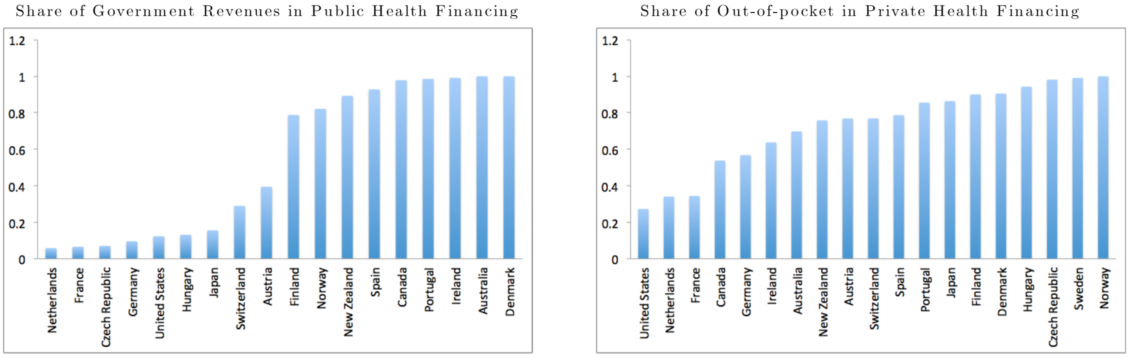

Figure 3: Composition of Private and Public Health Financing for a Sample of Democratic Countries

the mixture of health expenditure. An increase in the prices of private health care services would lead to a higher share of private health expenditure if the demand for private health care is relatively inelastic. Hence differences in the relative prices of health care services across countries, which are well observed in the data (see Gerdtham and et al (1992)), contribute to the observed differences in the mixture of health expenditure across countries.

4 Conclusions

Despite the large variations in the public-private mix of health expenditure across countries, factors that critically affect the composition of health expenditure have rarely been examined analytically and empirically in the existing literature. In this study, we examined, in the context of a simple overlapping generations model, how the public-private mix of health spending is determined through majority voting and how this decision is affected by various preference and economic factors. Further, we calibrated the model to conduct a quantitative exercise. The quantitative results are in line with the data, in particular, the predicted mixture of health expenditure matches the data reasonably well for a group of advanced democratic countries, suggesting that the model provides a promising framework to study the choice of public and private spending on health care.

The quantitative exercise also revealed the importance of the degree of substitutability between public and private health and the relative effectiveness of public health vs. private health in explaining the composition of health expenditure, and provided a way to infer these unobservable parameters from the data. Knowing about them can help policy makers design or reform health care policies to achieve the goals of efficiency and equity in health care financing.

As one of the first few attempts to formally examine the public-private mix of health expenditure, this study utilized a simple framework which incorporates voting in a dynamic macro-theoretic model. The model considered several important factors for decisions concerning public and private health spending. However, it abstracted from a few dimensions that are potentially important for the mixture of health spending, such as the pricing of public relative to private health services and the age-dependent demand for health care services. In addition, the substantial differences across countries in the inferred values for the degree of substitutability between public and private health and their relative effectiveness call for a further investigation into deeper forces underlying these differences, whether they are health care policies and regulations, or demographical and political factors. These are left for future research.

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Appendix

Proof of Second-Order Condition for Local Maximum. Recall that the functions for optimal private health expenditure, health capital, private savings and survival probability are given by $h_{i,t} \equiv h(y_{i,t}; \tau_t)$, $X_{i,t} \equiv X(y_{i,t}; \tau_t)$, $s_{i,t} \equiv s(y_{i,t}; \tau_t)$, and $p_{i,t} \equiv p(y_{i,t}; \tau_t)$, respectively.

Rewrite the first-order condition in (11) as

$$\Upsilon_{i,t} \equiv -\frac{1+\beta p_{i,t}}{((1-\tau_t)y_{i,t}-h_{i,t})} + \beta \frac{\partial p}{\partial h_{i,t}} \ln(R_{t+1}s_{i,t}) = 0. \tag{A.1}$$

By differentiating (A.1) with respect to $h_{i,t}$, we get:

$$\frac{\partial \Upsilon_{i,t}}{\partial h_{i,t}} = -\frac{\beta \frac{\partial p}{\partial h_{i,t}}((1-\tau_t)y_{i,t}-h_{i,t}) + (1+\beta p_{i,t})}{((1-\tau_t)y_{i,t}-h_{i,t})^2} + \beta \frac{\partial^2 p}{\partial h_{i,t}^2} \ln(R_{t+1}s_{i,t}) + \beta \frac{\partial p}{\partial h_{i,t}} \frac{\frac{\partial s}{\partial h_{i,t}}}{s_{i,t}}$$

where $\partial p/\partial h_{i,t} > 0$, $\ln(R_{t+1}s_{i,t}) > 0$ (since we assumed the utility from death is zero) and $\partial p^2/\partial h_{i,t}^2 < 0$. Thus, $\partial \Upsilon_{i,t}/\partial h_{i,t} < 0$ when $\partial s/\partial h_{i,t} < 0$. To ensure $\partial s/\partial h_{i,t} < 0$, note that from Eq. (9), we can obtain

$$\begin{aligned}
\frac{\partial s}{\partial h_{i,t}} &= \frac{\beta \frac{\partial p}{\partial X}\frac{\partial X}{\partial h_{i,t}}((1-\tau_t)y_{i,t}-h_{i,t})}{(1+\beta p_{i,t})^2} - \frac{\beta p_{i,t}}{(1+\beta p_{i,t})} \\
&= \frac{\beta p_{i,t}}{(1+\beta p_{i,t})}\left(\frac{p_{i,t}}{(1+\beta p_{i,t})}\frac{\left((1-\tau_t)\frac{y_{i,t}}{h_{i,t}}-1\right)}{\left(\frac{\phi_H}{\phi_h}\left(\frac{h_{i,t}}{H_t}\right)^{-\rho}+1\right)X_{i,t}} - 1\right)
\end{aligned}$$

Hence, $\partial s/\partial h_{i,t} < 0$ if and only if

$$\frac{p_{i,t}}{(1+\beta p_{i,t})}\frac{\left((1-\tau_t)\frac{y_{i,t}}{h_{i,t}}-1\right)}{\left(\frac{\phi_H}{\phi_h}\left(\frac{h_{i,t}}{H_t}\right)^{-\rho}+1\right)X_{i,t}} < 1 \tag{A.2}$$

As a result, when (A.2) holds, we obtain $\partial \Upsilon_{i,t}/\partial h_{i,t} < 0$, and so the second-order condition for local maximum is satisfied. ■

Proof of Proposition 1. (i) Using (A.1), we obtain

$$\frac{\partial h}{\partial y_{i,t}} = -\frac{\frac{\partial \Upsilon_{i,t}}{\partial y_{i,t}}}{\frac{\partial \Upsilon_{i,t}}{\partial h_{i,t}}},$$

where
$$\frac{\partial \Upsilon_{i,t}}{\partial y_{i,t}} = \frac{(1-\tau_t)}{((1-\tau_t)y_{i,t}-h_{i,t})}\left(\frac{(1+\beta p_{i,t})}{((1-\tau_t)y_{i,t}-h_{i,t})}+\beta\frac{\partial p}{\partial h_{i,t}}\right) > 0,$$

and $\partial\Upsilon_{i,t}/\partial h_{i,t} < 0$ when (A.2) holds; thus, $\partial h/\partial y_{i,t} > 0$.

(ii) Using (A.1), we obtain
$$\frac{\partial h}{\partial \tau_t} = -\frac{\frac{\partial \Upsilon_{i,t}}{\partial \tau_t}}{\frac{\partial \Upsilon_{i,t}}{\partial h_{i,t}}}, \tag{A.3}$$

where $\partial\Upsilon_{i,t}/\partial h_{i,t} < 0$ when (A.2) holds, and

$$\begin{aligned}\frac{\partial \Upsilon_{i,t}}{\partial \tau_t} &= -\frac{\beta\frac{\partial p}{\partial \tau_t}\left((1-\tau_t)y_{i,t}-h_{i,t}\right)+\left(1+\beta p_{i,t}\right)y_{i,t}}{\left((1-\tau_t)y_{i,t}-h_{i,t}\right)^2}+\beta\frac{\partial^2 p}{\partial h_{i,t}\partial\tau_t}\ln\left(R_{t+1}s_{i,t}\right) \\ &\quad +\beta\frac{\partial p}{\partial h_{i,t}}\frac{\frac{\partial s}{\partial \tau_t}}{s_{i,t}},\end{aligned} \tag{A.4}$$

where $\partial p/\partial\tau_t > 0$, and $\partial p/\partial h_{i,t} > 0$. Then, $\partial\Upsilon_{i,t}/\partial\tau_t < 0$ when $\partial^2 p/\partial h_{i,t}\partial\tau_t < 0$ and $\partial s/\partial\tau_t < 0$.

To ensure $\partial^2 p/\partial h_{i,t}\partial\tau_t < 0$,we first rewrite $\partial^2 p/\partial h_{i,t}\partial\tau_t$ as $\left(\partial^2 p/\partial h_{i,t}\partial H_t\right)\bar{y}_t$ since $\tau_t = H_t/\bar{y}$. Note that

$$\frac{\partial^2 p}{\partial h_{i,t}\partial H_t} = \frac{\phi_h h_{i,t}^{\rho-1}}{(1+\kappa X_{i,t})^2\left(\phi_h h_{i,t}^{\rho}+\phi_H H_t^{\rho}\right)}\left(\frac{\frac{\partial X}{\partial H_t}\left(1-\kappa X_{i,t}\right)}{(1+\kappa X_{i,t})}-\frac{X_{i,t}\rho\phi_H H_t^{\rho-1}}{\left(\phi_h h_{i,t}^{\rho}+\phi_H H_t^{\rho}\right)}\right)$$

Hence, $(\partial^2 p/\partial h_{i,t}\partial H_t) < 0$ if
$$1-\kappa X_{i,t} < 0 \tag{A.5}$$

Next, to ensure $\partial s/\partial\tau_t < 0$, from Eq. (9) we can obtain

$$\frac{\partial s}{\partial \tau_t} = \frac{\beta p_{i,t}}{(1+\beta p_{i,t})}\left(\frac{p_{i,t}}{(1+\beta p_{i,t})}\frac{\left((1-\tau_t)\frac{y_{i,t}}{H_t}-\frac{h_{i,t}}{H_{i,t}}\right)}{\left(\frac{\phi_h}{\phi_H}\left(\frac{h_{i,t}}{H_t}\right)^{\rho}+1\right)X_{i,t}}-1\right)$$

Thus, $\partial s/\partial \tau_t$ is negative if and only if

$$\frac{p_{i,t}}{(1+\beta p_{i,t})}\frac{\left((1-\tau_t)\frac{y_{i,t}}{H_t}-\frac{h_{i,t}}{H_{i,t}}\right)}{\left(\frac{\phi_h}{\phi_H}\left(\frac{h_{i,t}}{H_t}\right)^{\rho}+1\right)X_{i,t}}<1 \tag{A.6}$$

As a result, when (A.5) and (A.6) hold, then $\partial \Upsilon_{i,t}/\partial \tau_t<0$, and so $\partial h/\partial \tau_t<0$.

(iii) Given any tax rate, we obtain

$$\frac{\partial p}{\partial y_{i,t}}=\frac{\psi_{i,t}^{\frac{1}{\rho}-1}\phi_h h_{i,t}^{\rho-1}}{(1+\kappa X_{i,t})^2}\frac{\partial h}{\partial y_{i,t}}$$

by differentiating (7) with respect to $y_{i,t}$. Thus, $\partial p/\partial y_{i,t}>0$ when $\partial h/\partial y_{i,t}>0$ which is the case when (A.2) holds. ■

Proof of Proposition 2. First, we show $\partial \tau_{i,t}/\partial y_{i,t}<0$. Denote $\tau_{i,t}$ as the preferred tax rates by agent i when tax rates are endogenously determined, and rewrite (13) as

$$\Omega_{i,t}\equiv\frac{h_{i,t}}{H_{i,t}}-\left(\frac{\phi_h}{\phi_H}\frac{y_{i,t}}{\bar{y}_t}\right)^{\frac{1}{1-\rho}}=0.$$

Thus,

$$\frac{\partial \tau_{i,t}}{\partial y_{i,t}}=-\frac{\frac{\partial \boldsymbol{\Omega}_{i,t}}{\partial y_{i,t}}}{\frac{\partial \boldsymbol{\Omega}_{i,t}}{\partial \tau_{i,t}}},$$

where

$$\frac{\partial \Omega_{i,t}}{\partial \tau_{i,t}}=\frac{\frac{\partial h}{\partial \tau_{i,t}}H_{i,t}-h_{i,t}\frac{\partial H}{\partial \tau_{i,t}}}{H_{i,t}^2}<0$$

since $\partial H/\partial \tau_{i,t}>0$, and $\partial h/\partial \tau_{i,t}<0$ when (A.5) and (A.6) hold (see the proof of Proposition 1(ii) shown above). And

$$\begin{aligned}\frac{\partial \Omega_{i,t}}{\partial y_{i,t}} &= \frac{\frac{\partial h}{\partial y_{i,t}}H_{i,t}-h_{i,t}\frac{\partial H}{\partial y_{i,t}}}{H_{i,t}^2}-\frac{1}{1-\rho}\left(\frac{\phi_h}{\phi_H}\frac{y_{i,t}}{\bar{y}_t}\right)^{\frac{1}{1-\rho}-1}\frac{\phi_h}{\phi_H}\frac{1}{\bar{y}_t}\\ &= \frac{1}{H_{i,t}}\frac{h_{i,t}}{y_{i,t}}\left(\frac{\partial h}{\partial y_{i,t}}\frac{y_{i,t}}{h_{i,t}}-\frac{1}{1-\rho}\right)\end{aligned}$$

where $\partial h/\partial y_{i,t}>0$ when (A.2) holds. Thus, $\partial \Omega_{i,t}/\partial y_{i,t}<0$ if and only if $(\partial h/\partial y_{i,t})\,(y_{i,t}/h_{i,t})<$

$1/(1-\rho)$. As a result, $\partial\tau_{i,t}/\partial y_{i,t} < 0$ if and only if

$$\frac{\partial h}{\partial y_{i,t}}\frac{y_{i,t}}{h_{i,t}} < \frac{1}{1-\rho}. \tag{A.7}$$

Recall that $\partial h/\partial y_{i,t} = -\left(\partial\Upsilon_{i,t}/\partial y_{i,t}\right)/\left(\partial\Upsilon_{i,t}/\partial h_{i,t}\right)$ and the expressions for $\partial\Upsilon_{i,t}/\partial y_{i,t}$ and $\partial\Upsilon_{i,t}/\partial h_{i,t}$ are given before.

Next, we show that voters' preferences are single-peaked. We differentiate the indirect utility, $V_{i,t}$, in (12) with respect to tax rates, $\tau_{i,t}$, and obtain

$$\frac{\partial V_{i,t}}{\partial\tau_{i,t}} = -\frac{(1+\beta p_{i,t})\left(y_{i,t}+\frac{\partial h}{\partial\tau_{i,t}}\right)}{\left((1-\tau_{i,t})\,y_{i,t}-h_{i,t}\right)} + \beta\frac{\partial p}{\partial\tau_{i,t}}\ln\left[\frac{R_{t+1}\beta p_{i,t}((1-\tau_{i,t})\,y_{i,t}-h_{i,t})}{(1+\beta p_{i,t})}\right], \tag{A.8}$$

where

$$\frac{\partial p}{\partial\tau_{i,t}} = \frac{X_{i,t}[\phi_H H_{it}^{\rho-1}\bar{y}_t + \phi_h h_{it}^{\rho-1}\frac{\partial h}{\partial\tau_{i,t}}]}{\left(1+\kappa X_{i,t}\right)^2\psi_{i,t}}. \tag{A.9}$$

By substituting $H_{i,t} = \tau_{i,t}\bar{y}_t$, $X_{i,t} = (\phi_H H_{i,t}^{\rho} + \phi_h h_{i,t}^{\rho})^{1/\rho}$, and (11) into (A.8), we obtain

$$\frac{\partial V_{i,t}}{\partial\tau_{i,t}} = \frac{\beta\psi_{i,t}^{\frac{1}{\rho}-1}[\phi_H(\tau_{i,t}\bar{y}_t)^{\rho-1}\bar{y}_t - \phi_h h_{it}^{\rho-1}y_{i,t}]}{\left(1+\kappa X_{i,t}\right)^2}\ln\left[\frac{R_{t+1}\beta p_{i,t}((1-\tau_{i,t})\,y_{i,t}-h_{i,t})}{(1+\beta p_{i,t})}\right]. \tag{A.10}$$

We show the single-peakedness of preference under the following three conditions: (i) $\partial V_{i,t}/\partial\tau_{i,t} > 0$ when tax rates are low enough such that they are lower than the preferred tax rate, (ii) $\partial V_{i,t}/\partial\tau_{i,t} = 0$ at the preferred tax rate, and (iii) $\partial V_{i,t}/\partial\tau_{i,t} < 0$ when tax rates are high enough such that they are higher than the preferred tax rates.

(i) Using (A.10) with $\rho \in (0,1)$ and $\ln(R_{t+1}s_{i,t}) = \ln\left[R_{t+1}\beta p_{i,t}((1-\tau_{i,t})\,y_{i,t}-h_{i,t})/\left(1+\beta p_{i,t}\right)\right] > 0$, it can be shown that if tax rates are within a low enough range of values, then $\partial V_{i,t}/\partial\tau_{i,t} > 0$. That is, $\partial V_{i,t}/\partial\tau_{i,t} > 0$ if $0 < \tau_{i,t} < \widehat{\tau}_{i,t}$, where $\widehat{\tau}_{i,t} \equiv (\phi_H\bar{y}_t/\phi_h y_{i,t})^{\frac{1}{1-\rho}}\,h_{i,t}/\bar{y}_t$. This is because when tax rates are within this range of values, $\phi_H(\tau_{i,t}\bar{y}_t)^{\rho-1}\bar{y}_t - \phi_h h_{it}^{\rho-1}y_{i,t} > 0$ in (A.10) which thus give us $\partial V_{i,t}/\partial\tau_{i,t} > 0$.

When $\tau_{i,t} \to 0$, it can be shown that both $p_{i,t}$ and $h_{i,t}$ take positive finite values using (7) and (11), respectively. Thus, for $\ln(R_{t+1}s_{i,t}) = \ln\left[R_{t+1}\beta p_{i,t}((1-\tau_{i,t})\,y_{i,t}-h_{i,t})/\left(1+\beta p_{i,t}\right)\right] > 0$ and $\rho \in (0,1)$, it is obvious that $\partial V_{i,t}/\partial\tau_{i,t} \to +\infty$ in (A.10) when $\tau_{i,t} \to 0$. Thus, $\partial V_{i,t}/\partial\tau_{i,t} > 0$ if tax rates are low enough such that $0 < \tau_{i,t} < \widehat{\tau}_{i,t}$.

(ii) When $\ln(R_{t+1}s_{i,t}) = \ln\left[R_{t+1}\beta p_{i,t}((1-\tau_{i,t})\,y_{i,t}-h_{i,t})/\left(1+\beta p_{i,t}\right)\right] > 0$, it can be easily checked from (A.10) that the preferred tax rate is given by $\widehat{\tau}_{i,t}$, which is the implicit tax rate that solves $\partial V_{i,t}/\partial\tau_{i,t} = 0$.

(iii) Using (A.10) with $\rho \in (0,1)$, it can be shown that $\partial V_{i,t}/\partial\tau_{i,t} < 0$ if tax rates are within a high enough range of values but do not exceed an upper limit so that $\ln(R_{t+1}s_{i,t}) = \ln\left[R_{t+1}\beta p_{i,t}((1-\tau_{i,t})\,y_{i,t}-h_{i,t})/\left(1+\beta p_{i,t}\right)\right] > 0$ and $h_{i,t} > 0$. That is, $\partial V_{i,t}/\partial\tau_{i,t} < 0$ if

$\widehat{\tau}_{i,t} < \tau_{i,t} < \overline{\tau}_{i,t}$, where $\overline{\tau}_{i,t} \equiv 1-(1+\beta p_{i,t})/(R_{t+1}\beta p_{i,t} y_{i,t}) - h_{i,t}/y_{i,t}$, assuming $\widehat{\tau}_{i,t} < \overline{\tau}_{i,t}$. This is because when tax rates are within this range of values, $\phi_H(\tau_{i,t}\bar{y}_t)^{\rho-1}\bar{y}_t - \phi_h h_{it}^{\rho-1} y_{i,t} < 0$ and $\ln[R_{t+1}\beta p_{i,t}((1-\tau_{i,t}) y_{i,t} - h_{i,t})/(1+\beta p_{i,t})] > 0$ in (A.10) which thus give us $\partial V_{i,t}/\partial \tau_{i,t} < 0$.

Conditions (i)-(iii) show that voters' preferences are single-peaked such that the majority choice of tax rate exists. As the preferred tax rates of voters decrease with their incomes, the majority choice of the tax rate is simply the preferred tax rate of the young adult with median income, denoted as $\tau_{m,t}$, which is uniquely determined by Eq. (13) with $i = m$. ■

Existence of a Voting Equilibrium. Here, we provide a discussion on the existence of a voting equilibrium, i.e., the existence of a fixed point for average survival probability, $\bar{p}_t$. The model defines a mapping $T : (0,1) \to (0,1)$, which maps a $\bar{p}_t \in (0,1)$ to $\bar{p}_t^N \equiv T(\bar{p}_t) \in (0,1)$ (by definition, the $p_{i,t}$'s are greater than 0 and less than 1). We argue that T is continuous, downward slopping, and $T(\bar{p}_t) > \bar{p}_t$ if $\bar{p}_t$ is close to 0. These properties ensure the existence and uniqueness of an intersection between the T curve and the 45 degree line (as shown in Figure 4 below), i.e., the existence and uniqueness of a fixed point of T.

From the definition of the voting equilibrium, it is obvious that the mapping T is continuous. In what follows, we first argue that T is downward slopping. We consider a fall in $\bar{p}_t$, or equivalently, a rise in $R_{t+1} \equiv (1+r_{t+1})/\bar{p}_t$, and discuss its effect on public health spending, private health spending, and average survival probability.

The effect of a rise in R_{t+1} on public health spending

We can show that $\tau_{m,t}$ or public health spending rises with R_{t+1}, using the equation that implicitly determines $\tau_{m,t}$. Define

$$\Omega_{m,t} \equiv \frac{h(y_{m,t};\tau_{m,t})}{\tau_{m,t}\bar{y}_t} - \left(\frac{\phi_h}{\phi_H}\frac{y_{m,t}}{\bar{y}_t}\right)^{\frac{1}{1-\rho}} = 0. \tag{A.11}$$

we obtain

$$\frac{d\tau_{m,t}}{dR_{t+1}} = -\frac{\frac{\partial \mathbf{\Omega}_{m,t}}{\partial R_{t+1}}}{\frac{\partial \mathbf{\Omega}_{m,t}}{\partial \tau_{m,t}}} = -\frac{\frac{\partial h}{\partial R_{t+1}}\frac{1}{\tau_{m,t}\bar{y}_t}}{\frac{1}{\tau_{m,t}\bar{y}_t}\left[\frac{\partial h}{\partial \tau_{m,t}} - \frac{h(y_{m,t};\tau_{m,t})}{\tau_{m,t}}\right]} = -\frac{\frac{\partial h}{\partial R_{t+1}}}{\frac{\partial h}{\partial \tau_{m,t}} - \frac{h(y_{m,t};\tau_{m,t})}{\tau_{m,t}}}, \tag{A.12}$$

where $\partial h/\partial R_{t+1}$ and $\partial h/\partial \tau_{m,t}$ denote the partial derivatives of $h(y_{m,t};\tau_{m,t})$ with respect to R_{t+1} and $\tau_{m,t}$, respectively. We have already established that the optimal private health spending decreases with tax rate in Proposition 1, so $\partial h/\partial \tau_{m,t} < 0$. It is also easy to show the optimal private health spending increases with R_{t+1} (when holding the tax rate constant), using the equation that implicitly determines the optimal private health spending (equation (11)), i.e., $\partial h/\partial R_{t+1} > 0$. Hence, $d\tau_{m,t}/dR_{t+1} > 0$.

The effect of a rise in R_{t+1} on private health spending

First, we can see from (A.11) that the private health spending of the median voter increases with R_{t+1}: a rise in R_{t+1} leads to an increase in $\tau_{m,t}\bar{y}_t$, so $h(y_{m,t};\tau_{m,t})$ has to increase as well to make (A.11) hold.

The effect of R_{t+1} on the private health spending of any agent i, $h(y_{i,t};\tau_{m,t})$, is given by

$$\frac{dh}{dR_{t+1}} = \frac{\partial h}{\partial R_{t+1}} + \frac{\partial h}{\partial \tau_{m,t}} \frac{d\tau_{m,t}}{dR_{t+1}}. \tag{A.13}$$

So there are two effects working together: an income effect as indicated by $\partial h/\partial R_{t+1}$ and a substitution effect as indicated by $(\partial h/\partial \tau_{m,t})(d\tau_{m,t}/dR_{t+1})$. Note that the income effect is positive, and the intuition is that agents are more affordable to invest in their health when R_{t+1} rises. The substitution effect is negative, and the intuition is that a higher R_{t+1} raises public health spending which tends to crowd out private health spending. Thus, whether the private health spending of agent i increases or decreases with R_{t+1} depends on whether the income effect dominates or the substitution effect dominates, which may depend on the values of ρ and ϕ_H as well as the income level of agent i.

The effect of a rise in R_{t+1} on survival probabilities

The survival probability of agent i, $p_{i,t}$, is a function of public health spending and agent i's private health spending. The effect of R_{t+1} on individual's survival probability is given by:

$$\frac{dp_{i,t}}{dR_{t+1}} = \frac{\frac{dX_{i,t}}{dR_{t+1}}}{(1+\kappa X_{i,t})^2}$$

where

$$\frac{dX_{i,t}}{dR_{t+1}} = X_{i,t}^{(1-\rho)} \left[\bar{y}_t \phi_H H_{m,t}^{\rho-1} \left(\frac{d\tau_{m,t}}{dR_{t+1}} \right) + \phi_h [h(y_{i,t};\tau_{m,t})]^{\rho-1} \left(\frac{dh}{dR_{t+1}} \right) \right] \tag{A.14}$$

and $d\tau_{m,t}/dR_{t+1}$ and dh/dR_{t+1} are given by (A.12) and (A.13), respectively. So, the sign of $dp_{i,t}/dR_{t+1}$ agrees with the sign of $dX_{i,t}/dR_{t+1}$. As discussed above, $d\tau_{m,t}/dR_{t+1} > 0$ but the sign of dh/dR_{t+1} is uncertain. Thus, the sign of $dX_{i,t}/dR_{t+1}$ is uncertain. We can see from (A.14) that X_{it} and hence $p_{i,t}$ increases with R_{t+1} if either of the following two cases hold: (i) the positive income effect dominates the negative substitution effect such that private health spending also increases with R_{t+1}; (ii) the substitution effect dominates such that private health spending decreases with R_{t+1}, however, the positive effect of R_{t+1} on public health spending dominates its overall effect on health status and survival probability.

Using (A.13) and $\tau_{m,t} = H_{m,t}/\bar{y}_t$, we can rewrite (A.14) as:

$$\frac{dX_{i,t}}{dR_{t+1}} = X_{i,t}^{(1-\rho)} \left[\bar{y}_t \left(\phi_H H_{m,t}^{\rho-1} + \phi_h [h(y_{i,t};\tau_{m,t})]^{\rho-1} \frac{\partial h}{\partial H_{m,t}} \right) \frac{d\tau_{m,t}}{dR_{t+1}} + \phi_h [h(y_{i,t};\tau_{m,t})]^{\rho-1} \frac{\partial h}{\partial R_{t+1}} \right],$$

where $\partial h/\partial H_{m,t} = (\partial h/\partial \tau_{m,t})/\bar{y}_t < 0$. Since $d\tau_{m,t}/dR_{t+1}$ and $\partial h/\partial R_{t+1}$ are both positive, a sufficient condition for $dX_{i,t}/dR_{t+1} > 0$ is that $(-\partial h/\partial H_{m,t}) \leq (\phi_H/\phi_h)\,[h(y_{i,t};\tau_{m,t})/H_{m,t}]^{1-\rho}$ (such that $\phi_H H_{m,t}^{\rho-1} + \phi_h [h(y_{i,t};\tau_{m,t})]^{\rho-1}(\partial h/\partial H_{m,t}) \geq 0$). That is, X_{it} or $p_{i,t}$ increases with R_{t+1} if the crowding out effect of public health spending on private health spending, as indicated by $(-\partial h/\partial H_{m,t})$, is sufficiently small. Further notice that by (A.11) and the property that $h(y_{i,t};\tau_{m,t})$ is strictly increasing with $y_{i,t}$, we have $(\phi_H/\phi_h)\left[h(y_{i,t};\tau_{m,t})/H_{m,t}\right]^{1-\rho} \geq (y_{m,t}/\bar{y}_t)$ for all agents with income level higher than the median income (with equality if $i=m$). So if $(-\partial h/\partial H_{m,t})$ is smaller than the income inequality measure $y_{m,t}/\bar{y}_t$, the sufficient condition would hold for all agents with income level higher than the median income.

Next, we conduct a numerical exercise to explore how public health spending, private health spending, individual survival probabilities and average survival probability depend on values of ρ and ϕ_H. Using baseline parameter values for Canada (see Table 1)[21], we vary the values of ρ and ϕ_H to see how public health spending, private health spending for different income draws, and survival probabilities change when we let R_{t+1} increase from a lower value to a higher value. Our numerical exercise confirms that public health spending always increases with the increase in R_{t+1}. It reveals that the effect on private health spending does depend on the values of ρ and ϕ_H and individual income levels. For low values of ρ, private health spending increases with the increase in R_{t+1} for all income draws, regardless of what value ϕ_H takes. The intuition for this result is that a low value of ρ implies that the elasticity of substitution between public and private health spending is low so that the negative substitution effect is small, and hence the positive income effect tends to dominate. For high values of ρ, private health spending tends to decrease (increase) with the increase in R_{t+1} for low (high) income draws. This tendency is stronger for higher values of ϕ_H, that is, with a higher ϕ_H value, more low income people decrease private health spending as R_{t+1} increases. These results are intuitive, as the substitution effect tends to be higher when ρ is high, public health is more effective (ϕ_H is high), and income is lower.

However, for a large number of values of ρ and ϕ_H considered, we always obtain higher individual survival probabilities for all income draws and hence a higher average survival probability, as R_{t+1} increases. This is because at all income draws, either both public and private health spending increase or the positive effect on survival probabilities from higher public health expenditure dominates even though private health spending decreases. In other words, there seems to exist a robust positive relationship between R_{t+1} and average survival probability.

Existence of a fixed point for average survival probability

The discussion above suggests that the mapping T is downward slopping. We now argue that $T(\bar{p}_t) > \bar{p}_t$ if $\bar{p}_t$ is very close to zero. When $\bar{p}_t$ is close to zero, R_{t+1} is very big. Because public health spending increases with R_{t+1}, as discussed above, the public health spending corresponding to a very large R_{t+1} is expected to be large. So individual survival probabilities should be significantly greater than zero, regardless of how much private health spending individuals choose. As a result, the average survival probability should be significantly greater than zero, i.e., larger than the $\bar{p}_t$ initially taken as given.

Figure 4 plots the T curve under the baseline calibration.[22] It has a unique intersection

[21] The parameter values are: $\beta = 0.4753$, $\kappa = 1.1784$, $\rho = 0.7936$, $\phi_H = 0.5267$, and the log-normal income distribution is the same as in the revised manuscript.

[22] The T curve is downward slopping but quite flat, with $T(\bar{p}_t)$ varying within a small range of 0.7 to 0.8 for $\bar{p}_t \in (0,1)$. This results from a high level of public health spending relative to average private health spending for any given $\bar{p}_t$ (public health accounts for 70 percent of total health spending in the baseline calibration).

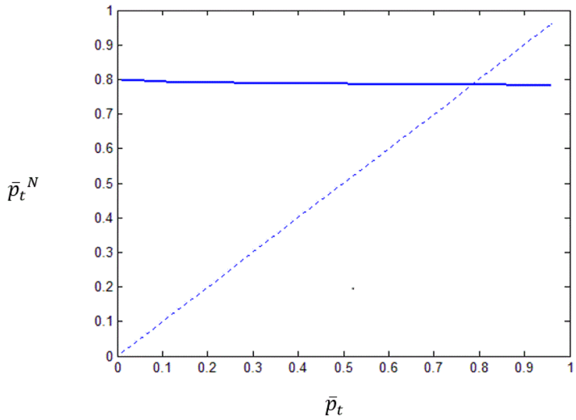

Figure 4: Existence of a fixed point for average survival probability

with the 45 degree line, i.e., T has a unique fixed point, suggesting that there exists a unique voting equilibrium. This is also verified in our quantitative analysis. When we numerically solve the model, we iterate on values of $\bar{p}_t$ until it converges, and the numerical computation suggests that the equilibrium $\bar{p}_t$ exists and it is unique. ∎